

Objectives

- We aim to reduce the communication cost of decentralized online convex optimization algorithms by using event-triggering.
- Energy saving is critical for battlefield applications, where the mobile devices of the soldiers have limited battery capacity and are not easy to recharge.

Approach

- Consider a network of m agents. At each time t , each agent i has a local loss function $f_{i,t}:\Omega\rightarrow\mathbb{R}$ and chooses an action $\mathbf{x}_{i,t}\in\Omega$. The global loss is $f_t(\mathbf{x})=\sum_i f_{i,t}(\mathbf{x})$, which all agents aim to minimize.
- To solve the decentralized online optimization problem, agents need to exchange information with neighbors. To reduce the communication cost, we use event-triggering and each agent sends its action to neighbors only when the innovation is greater than some threshold τ_t .

Algorithm

Compute the new action as

$$\mathbf{x}_{i,t+1} = \Pi_{\Omega} \left(\sum_{j=1}^m a_{ij} \hat{\mathbf{x}}_{j,t} - \eta_t \nabla f_{i,t}(\hat{\mathbf{x}}_{i,t}) \right).$$

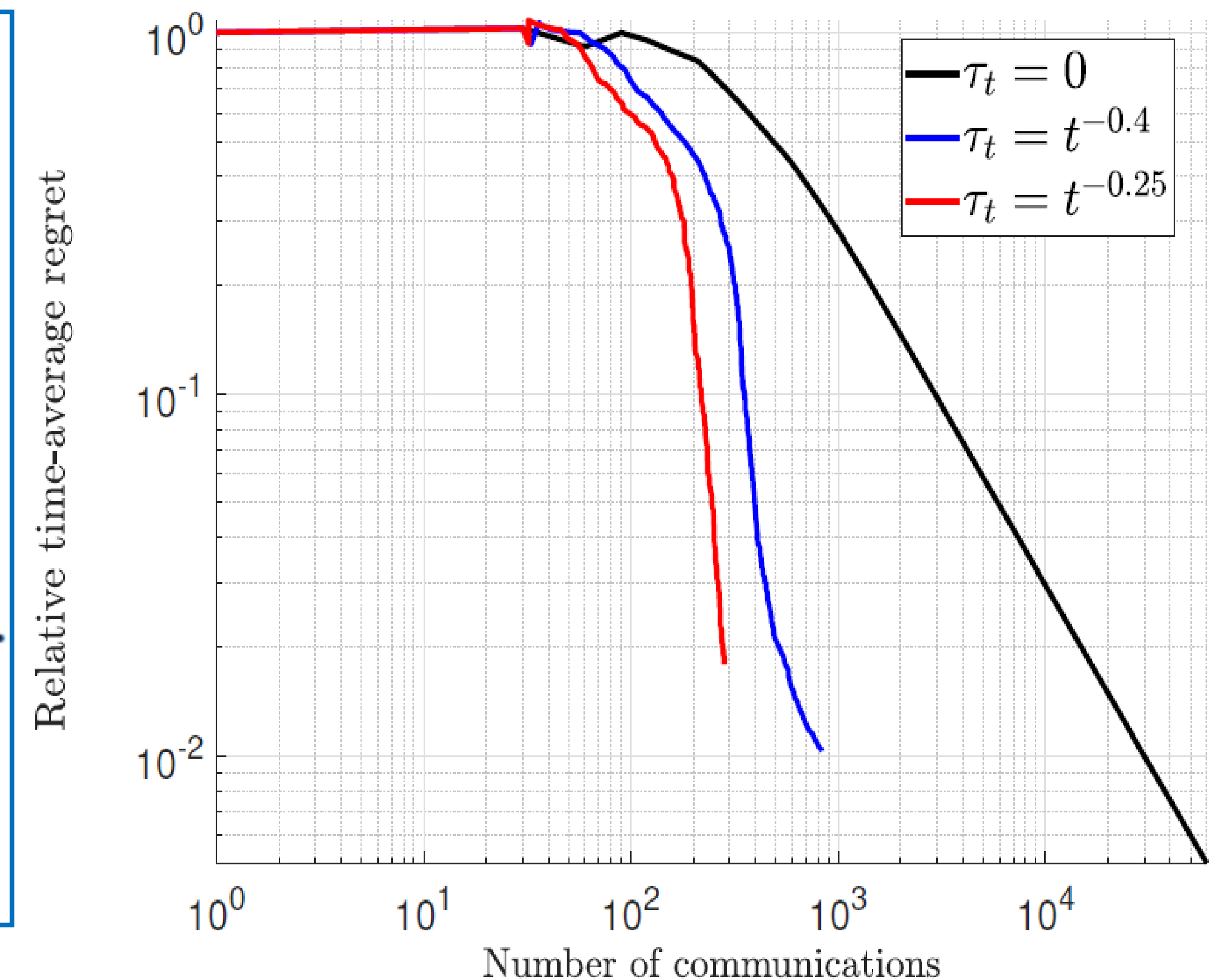
if $\|\mathbf{x}_{i,t+1} - \hat{\mathbf{x}}_{i,t}\| \geq \tau_{t+1}$, then

Send $\mathbf{x}_{i,t+1}$ to neighbors \mathcal{N}_i . Set $\hat{\mathbf{x}}_{i,t+1} = \mathbf{x}_{i,t+1}$.

else (i.e., $\|\mathbf{x}_{i,t+1} - \hat{\mathbf{x}}_{i,t}\| < \tau_{t+1}$)

Do not send and set $\hat{\mathbf{x}}_{i,t+1} = \hat{\mathbf{x}}_{i,t}$.

end if



Results

Theorem. Under standard assumptions, we have

$$\sum_{t=1}^T [f_t(\mathbf{x}_{j,t}) - f_t(\mathbf{x}^*)] \leq \mathcal{O} \left(\sqrt{T \left(1 + \sum_{t=2}^T \tau_t \right)} \right).$$

$$\mathbf{x}^* \in \arg \min_{\mathbf{x} \in \Omega} \sum_{t=1}^T f_t(\mathbf{x}).$$

- If $\sum_{t=2}^T \tau_t$ is sublinear, so is the regret of every agent.
- Tradeoff between communication cost and optimization performance

Conclusion

Event-triggering can save the communication cost by one to two orders of magnitude while achieving the same optimization performance.

Path Forward

Ascertaining the relation between the communication cost and event-triggering thresholds, we can devise the optimal event-triggering schemes and achieve the best optimization performance within a communication budget.

Publication

Xuanyu Cao and Tamer Başar, "Decentralized online convex optimization with event-triggered communications", *IEEE Transactions on Signal Processing*, vol. 69, pp. 284-299, 2021.